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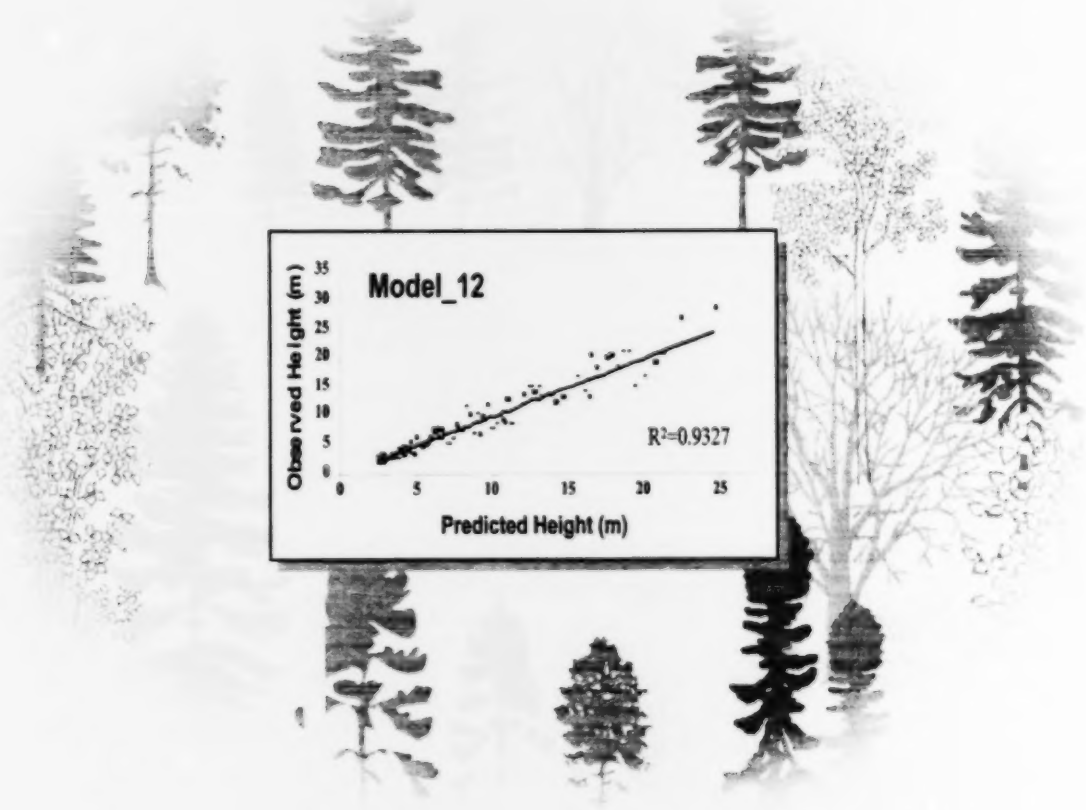
Nonlinear Height-Diameter Models for Nine Boreal Forest Tree Species in Ontario



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Nonlinear Height-Diameter Models for Nine Boreal Forest Tree Species in Ontario

by

Changhui Peng

1999

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Abstract

Twenty-five nonlinear height–diameter models were fitted and developed for 9 forest species in Ontario's boreal forests based on individual tree height and diameter data (n= 21,571) collected from permanent sample plots (PSPs) across northern Ontario. Available tree height and diameter data for each species were split into 2 data sets: the majority of the data (90%) were used for model development and the remaining data (10%) were reserved for model validation. Comparison of mean square error and R^2 values show that most concave downward and sigmoidal equations capture the height –diameter relationships for Ontario tree species. Validation of 6 selected models using independent data sets suggests that sigmoidal equations such as the Chapman–Richards, Weibull–type, and Schnute equations provide the best height predictions. However, using these models to extrapolate beyond the range of available data may increase the error margin for large trees.

Key Words: growth function, model validation, predicted error, permanent sample plot

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Contents

Introduction **1**

Data, Models and Methods **1**

Results and Discussion **3**

Model Development 3

Model Validation 3

Conclusions **4**

Tables **5**

TABLE 1. Common names, scientific names and assigned codes for the species included in this work. 5

TABLE 2. Summary statistics of diameter at breast height (DBH) outside bark and total tree height (HT) for northern Ontario data used to fit the models. 5

TABLE 3. Summary statistics of diameter at breast height (DBH) outside bark and total tree height (HT) for northern Ontario data used to validate the models. 6

TABLE 4. Nonlinear height-diameter models selected for fitting using data from boreal forests in northern Ontario. 7

TABLE 5. Parameter estimates for height-diameter models for jack pine 8

TABLE 6. Parameter estimates for height-diameter models for black spruce 8

TABLE 7. Parameter estimates for height-diameter models for white spruce 9

TABLE 8. Parameter estimates for height-diameter models for trembling aspen 9

TABLE 9. Parameter estimates for height-diameter models for white pine 10

TABLE 10. Parameter estimates for height-diameter models for red pine 10

TABLE 11. Parameter estimates for height-diameter models for balsam fir 11

TABLE 12. Parameter estimates for height-diameter models for yellow birch 11

TABLE 13: Parameter estimates for height-diameter models for balsam poplar 12

Figures **13**

FIGURE 1. Total height (HT) plotted against diameter at breast (DBH) for jack pine. 13

FIGURE 2. Observed vs. predicted tree heights for the validation data set for jack pine. 14

FIGURE 3. Observed vs. predicted tree heights for the validation data set for black spruce 15

FIGURE 4. Observed vs. predicted tree heights for the validation data set for white spruce. 16

FIGURE 5. Observed vs. predicted tree heights for the validation data set for trembling aspen. 17

FIGURE 6. Observed vs. predicted tree heights for the validation data set for white pine. 18

FIGURE 7. Observed vs. predicted tree heights for the validation data set for red pine. 19

FIGURE 8. Observed vs. predicted tree heights for the validation data set for balsam fir. 20

FIGURE 9. Observed vs. predicted tree heights for the validation data set for yellow birch. 21

FIGURE 10. Observed vs. predicted tree heights for the validation data set for balsam poplar. 22

FIGURE 11. Average prediction errors from 6 tree height-diameter models [12], [13], [15], [18], [19] and [22] for the 5-cm DBH classes for 9 tree species in Ontario's boreal forests. 23

FIGURE 12. Average mean standard deviation of prediction errors from 6 tree height-diameter models [12], [13], [15], [18], [19] and [22] for the 5-cm DBH classes for 9 tree species in Ontario's boreal forests. 25

References **26**

Introduction

Individual-tree height and diameter are essential forest inventory measurements for estimating timber volume and site index and are also important variables in growth and yield modelling. The Ontario Forest Growth and Yield Program (OMNR 1997) identified an urgent need to produce local tree height-diameter equations for estimating tree volume. Forest resource managers require tree volume information to produce yield estimates for timber inventory and improve forest management decision-making.

The relationship between tree heights and diameters is one of the most important elements of forest structure. Estimating individual tree volume and site index, and describing stand growth dynamics and succession over time requires accurate height–diameter models (Curtis 1967; Botkin et al. 1972). A number of height-diameter equations have been developed for various tree species in north America (e.g. Curtis 1967; Wykoff et al. 1982; Larsen and Hann 1987; Wang and Hann 1988; Huang et al. 1992, Moor et al. 1996; Zhang 1997; Lappi 1997). In practice, these tree height-diameter equations can be used to predict the "missing" heights from field measurement of tree diameters (Larsen and Hann 1987), and to estimate individual tree biomass using appropriate single–tree biomass equations (Singh 1982; Penner et al. 1997). In forest inventory, total tree height is often estimated from observed tree diameter at breast height (DBH) outside bark. Tree diameter can easily be measured at low cost. But tree height data are relatively more difficult and costly to collect. Thus models based solely on diameter measurements are most cost effective.

The primary objectives of this work were to (1) develop nonlinear height-diameter models for 9 boreal forest tree species in Ontario, (2) evaluate the relative performance of these models calibrated for a range of site productivity and tree size, and (3) select good model candidates for further validation and volume estimation in this region.

Data, Models and Methods

Individual tree height-diameter data (n=22,571) for 9 boreal forest species were collected from PSPs across northern Ontario (Hayden et al. 1995). Common names, scientific names, and assigned codes are listed for each species (Table 1). All sampled trees were measured for diameter at breast height (DBH) outside bark, and total height (HT). Forked or top damaged trees were excluded from the analysis. Available tree height-diameter data were divided into 2 data sets. Following Moore et al. (1996), the majority of the data (90%) were used for model development. Ten percent of the trees were systematically selected across the range of diameter for each species and reserved for model validation. For example, data from 4954 jack pine trees were selected a from model fitting. The remaining 550 trees were used for model validation. These data sets

are shown in Figure 1. Summary statistics for height and diameters are provided for all 9 species in Tables 2 and 3.

To date, many nonlinear models have been used for modelling tree height-diameter relationships (e.g., Huang et al. 1992; Moore et al. 1996; Zhang 1997; Fang and Bailey 1998). Height-diameter models were selected by examining height-diameter relationships revealed by plotting HT against DBH for 9 individual species. The scatter plots of tree HT versus DBH present typical sigmoidal-concave curves for all species. A complete list of the selected models is shown in Table 4. They include those used by Curtis (1967), Huang et al. (1992), Arabatzis and Burkhart (1992), Moore et al. (1996), Zhang (1997), and Fang and Bailey (1998). Polynomial-type height-diameter models were excluded from this study because extrapolation of such models often leads to unrealistic height predictions (Huang et al. 1992, Huang 1999).

The 25 height-diameter functions (Table 4) were fitted to all available tree height-diameter data for each species. Parameters were estimated using the PROC NLIN procedure in SAS (Statistical Analysis System) (SAS Institute Inc. 1990). I used the Marquardt method because it is considered to be most useful when the parameter estimates are highly correlated (Fang and Bailey 1998). To ensure that the solution is global rather than a local least–squares solution, multiple initial values of model parameters were provided for fitting. The validity of least-squares assumptions was investigated. No significant evidence of unequal error variances, as has been observed in other studies (e.g., Huang et al. 1992), was found. Therefore, ordinary non–linear least–squares rather than weighted regression were used to estimate parameters. Each model was evaluated by the mean square error (MSE) and R^2 value of the model. The MSE and R^2 were calculated using the equations:

$$\mathbf{MSE} = \sum_{i=1}^{n} (H_i - \hat{H}_i)^2 / n$$

$$\mathbf{R^2} = 1 - [\sum_{i=1}^{n} (H_i - \hat{H}_i)^2 / \sum_{i=1}^{n} (H_i - \bar{H}_i)^2]$$

Where H_i is the observed and $\hat{H}_i$ is the predicted height for the *i*th tree, $\bar{H}_i$ is the observed mean tree height, and n is the number of observations. For any appropriately fitted height–diameter models, MSE should be small and R^2 should be large. A higher R^2 value indicates a better goodness-of-prediction for the data set.

The model validation data (Table 3) were divided into 8 DBH classes (e.g., <5 cm, 5–10 cm, 10–15 cm, 15–20 cm, 20–25 cm, 25–30 cm, 30–35 cm, and >35 cm) for jack pine (JP), trembling aspen (TA), white pine (WP) and red pine (RP); and 6 DBH classes (e.g., <5 cm, 5–10 cm, 10–15 cm, 15–20 cm, 20–25 cm, and >25 cm) for the remaining species. Predicted error (E_i) is calculated as the difference between observed tree height ($H_{obs\,(i)}$) and predicted tree height ($H_{pre(i)}$):

$$E_i = H_{obs(i)} - H_{pre(i)}$$

Positive prediction errors indicate underestimation, while negative errors indicate overestimation.

Results and Discussion

Model Development

The parameter estimates, mean square error (MSE) and R^2 value for each model are provided by species (Tables 5-13). The highest mean R^2 value was found in fitting height-diameter models for balsam poplar, and the lowest average MSE was observed for balsam fir, which has a smaller mean DBH than the other species. The asymptote coefficients (coefficient *a* in Tables 5-13) produced by 25 models using the same data sets were variable with the exception of models [12]* and [13] and models [17] and [18], which had similar asymptotes for all species.

Model statistics suggested that models [12], [13], [15], [18], [19], and [22] were equally well fitted to the tree height–diameter data of most of the 9 species (Tables 5–13), which is consistent with the findings reported by Huang et al. (1992) for major Alberta tree species, and by Zhang (1997) for 10 tree species in the inland northwest of the United States. All model coefficients were statistically significant at a = 0.05 (not shown). Each of these 6 models explained at least 96% of the total variation in tree heights. Models [12] (Richards), [13] (Weibull) and [15] (Schnute) had relatively smaller MSE than the other 3 models for all species except that models [18] (Exponential) and [19] (Modified logistic function) fitted the black spruce (BS), white pine (WP) and balsam fir (BF) data best. Mean values of MSE and R^2 ranged from 2.39 to 11.44, and 0.96 to 0.99, respectively. But the differences in MSE among these models for individual tree species were not significant.

In the literature, these 6 nonlinear growth functions are often selected as good candidate height–diameter models for most tree species. They not only have appropriate mathematical features and the potential for biological interpretation of parameters, but also provide reasonable predictions of tree height–diameter relationships (Brewer et al. 1985; Arabatzis and Burkhart 1992; Huang et al. 1992, Zeide 1993; Zhang 1997; Fang and Bailey 1998; Huang 1999). The results presented in this study, as well as those reported by Huang et al. (1992) and Zhang (1997), support these conclusions. However, these provincially based height–diameter models do not account for differences among ecological site regions, and are appropriate for making height predictions on a provincial basis only. Further development of ecoregion–based, individual-tree height–diameter models is of critical importance for ecosystem-based forest management (Huang 1999; Huang et al. 1999).

Model Validation

Comparisons of heights predicted by 6 models with heights observed in the validation data set (Table 3) for individual species are provided in Figures 2-10. The mean coefficient of determination (R^2) averaged over 6 models is about 0.85 for jack pine (JP), 0.88 for black spruce (BS), 0.93 for white spruce (WS), 0.89 for trembling aspen (TA), 0.86 for white pine (WP), 0.82 for red pine (RP), 0.90 for balsam fir (BF), 0.94 for yellow birch (YB), and 0.87 for balsam poplar (BP), respectively. However, the

* Numbers in square brackets refer to model numbers in Table 4.

difference in R^2 among the 6 models for the same species is small. Figure 11 illustrates the mean predicted error for 5-cm DBH classes and overall mean predicted error across the DBH class for each model and tree species. In general, overall mean standard deviations of errors predicted by the 6 models ranged from 1.3 to 3.5 m depending upon tree species, with the smallest errors in the 0 to 5 cm class for all species (Figure 12). All 6 models produced similar small prediction errors (<1 m) for small trees (DBH <20 cm), with model [22] having the largest errors. However, all the models underestimated heights of large trees for yellow birch, white spruce, trembling aspen, balsam poplar, white pine, black spruce, and balsam fir, and overestimated the heights of large trees for jack pine and red pine. Among these models, model [22] produced the largest mean predicted errors. Models [12], [13], and [15] were the best prediction of height for all species. However, using the models to extrapolate beyond the data range may increase the degree of over- or underestimation for large trees (Zhang et al. 1996). Maurer (1995) reported that Schnumacher's 1939 model underestimated average height of jack pine and black spruce in northeastern Ontario for very large diameter trees by up to 25%.

Conclusions

Analysis of 25 nonlinear height–diameter models fitted for 9 boreal forest tree species shows that most concave and sigmoidal functions are able to describe tree height–diameter relationships in northern Ontario. Model statistics suggest that models such as the Chapman–Richards, Weibull–type, Schnute, Exponential and Korf/Lundqvist were equally well suited to tree height–diameter data of 9 species in northern Ontario, which is consistent with the findings reported by Huang et al. (1992) for major Alberta tree species, and by Zhang (1997) for 10 tree species in the inland northwest of the United States. Validation of 6 selected models using independent data sets indicates that sigmoidal equations such as the Chapman–Richards, Weibull–type, and Schnute equations provide the most satisfactory results. However, extrapolating these models beyond the range of the calibration data may increase predicted errors for large trees.

Tables

TABLE 1. Common names, scientific names, and assigned codes for the species included in this work.

Common Name	Scientific Name	Code
Jack pine	*Pinus banskiana*	JP
Black spruce	*Picea mariana*	BS
White spruce	*Picea glauca*	WS
Trembling aspen	*Populus tremuloides*	TA
White pine	*Pinus strobus*	WP
Red pine	*Pinus resinosa*	RP
Balsam fir	*Abies balsamea*	BF
Yellow birch	*Betula alleghaniensis*	YB
Balsam poplar	*Populus balsamifera*	BP

TABLE 2. Summary statistics of diameter at breast height (DBH) outside bark and total tree height (HT) for northern Ontario data used to fit the models.

Species	Number of trees	DBH (cm)				HT (m)			
		Mean	Min.	Max.	Std. Dev.	Mean	Min.	Max.	Std. Dev.
Jack pine	4954	14.79	1.40	44.80	7.91	13.28	2.06	28.02	6.48
Black spruce	5555	11.31	0.70	36.50	6.10	10.24	1.42	26.80	4.92
White spruce	743	12.93	2.50	56.30	9.63	9.45	2.12	30.78	6.06
Trembling aspen	3089	17.19	2.50	55.50	10.32	16.38	2.75	35.00	6.91
White pine	2162	21.48	2.50	90.20	14.41	15.09	1.49	38.87	7.94
Red pine	1332	22.03	2.50	61.20	12.70	16.26	1.97	40.68	8.43
Balsam fir	1845	8.09	2.40	42.70	5.43	7.22	1.52	26.88	4.24
Yellow birch	398	15.07	2.50	77.20	13.01	13.41	3.61	26.22	5.69
Balsam poplar	240	23.36	2.60	55.10	11.11	19.48	2.21	32.38	6.42

TABLE 3. Summary statistics of diameter at breast height (DBH) outside bark and total tree height (HT) for northern Ontario data used to validate the models.

Species	Number of trees	DBH (cm) Mean	Min.	Max.	Std. Dev.	HT (m) Mean	Min.	Max.	Std. Dev.
Jack pine	550	14.81	2.50	42.10	7.92	13.32	2.15	24.96	6.42
Black spruce	616	11.34	2.10	33.30	6.12	10.25	1.30	24.80	4.93
White spruce	82	12.99	2.50	48.10	9.66	9.82	2.30	28.97	6.38
Trembling aspen	343	17.24	2.50	50.90	10.36	16.47	3.40	33.94	6.77
White pine	240	21.59	2.50	78.20	14.55	15.05	2.74	31.90	7.95
Red pine	148	22.56	2.60	68.50	22.56	16.05	2.09	36.38	8.16
Balsam fir	204	8.03	2.50	27.80	5.26	7.17	1.80	22.95	4.24
Yellow birch	44	15.64	2.50	66.70	14.04	13.76	3.27	27.50	6.03
Balsam poplar	26	23.53	4.00	45.30	10.70	20.18	5.59	29.02	6.03

TABLE 4. Nonlinear height-diameter models selected for fitting using data from boreal forests in northern Ontario.

Model number and form*	References
Two-parameter models:	
[1] $HT = 1.3 + aD^{b}$	Stoffels and van Soest, 1953; Stage 1975
[2] $HT = 1.3 + e^{a+b/(D+1)}$	Schreuder et al. 1979
[3] $HT = 1.3 + aD/(b+D)$	Wykoff et al. 1982
[4] $HT = 1.3 + a(1-e^{-bD})$	Bates and Watts 1980; Ratkowsky 1990
[5] $HT = 1.3 + D^2/(a+bD)^2$	Meyer 1940; Farr et al. 1989; Moffat et al. 1991
[6] $HT = 1.3 + a\,e^{b/D}$	Loetsch et al. 1973
[7] $HT = 1.3 + 10^{a}\,D^{b}$	Burkhart and Strub 1974; Buford 1986
[8] $HT = 1.3 + aD/(D+1) + bD$	Larson 1986; Watts 1983
[9] $HT = 1.3 + a(D/(1+D))^{b}$	Curtis 1967; Prodan 1968
Three-parameter models:	
[11] $HT = 1.3 + a/(1+be^{-cD})$	Pearl and Reed 1920
[12] $HT = 1.3 + a(1- e^{-bD^{c}})$	Richards 1959
[13] $HT = 1.3 + a(1- e^{-bD^{c}})$	Yang et al. 1978
[14] $HT = 1.3 + a\,e^{-be^{-cD}}$	Winsor 1932
[15] $HT = \{y_1^b + (c^b - y_1^b)[1-e^{-a(D-D_0)}]/[1-e^{-a(D_2-D_0)}]\}^{1/b}$	Schnute, 1981
[16] $HT = 1.3 + D^2/(a + bD + cD^2)$	Curtis 1967; Prodan 1968
[17] $HT = 1.3 + a\,D^{bD^{-c}}$	Sibbesen 1981
[18] $HT = 1.3 + a\,e^{b/(D+c)}$	Ratkowsky 1990
[19] $HT = 1.3 + a/(1+b^{-1}D^{-c})$	Ratkowsky and Reedy 1986
[21] $HT = 1.3 + a + b/(D+c)$	Tang 1994
[22] $HT = 1.3 + a\,e^{(-bD^{-c})}$	Stage 1963; Zeide 1989
[24] $HT = 1.3 + a[e^{-e^{(-b(D-c))}}]$	Seber and Wild 1989
Four-parameter models:	
[20] $HT = 1.3 + a(1-be^{-cD})^{d}$	Richards 1959
[23] $HT = 1.3 + a[e^{-be^{-cD^{d}}}]$	Zeide 1993
[25] $HT = 1.3 + a\{1-e^{[-b(D-c)^{d}]}\}$	Seber and Wild 1989

* HT, total tree height (m); D, DBH (cm); *a,b,c,d*, parameters to be estimated; e, base of the natural logarithm (˜ 2.71828); 1.3 is a constant used because DBH is measured at 1.3 m aboveground. For eq. 15: $y_1 = 1.3$, $D_0 = 0.0$, $D_2 = 100.0$.

TABLE 5. Parameter estimates for height-diameter models for jack pine.

Model	a	b	c	d	MSE	R^2
[1]	1.3065	0.8346			7.7543	0.9582
[2]	3.4815	-14.0453			6.6504	0.9642
[3]	69.2348	65.8852			7.2715	0.9608
[4]	38.9099	0.0268			7.2087	0.9612
[5]	3.0239	0.2771			6.6564	0.9641
[6]	30.6318	-12.1256			6.8016	0.9634
[7]	0.1161	0.8346			7.7543	0.9582
[8]	1.6027	0.7141			8.1936	0.9558
[9]	31.5160	13.0399			6.7186	0.9638
[10]	3.7595	-7.8408	-0.7033		6.6786	0.9640
[11]	19.9590	13.2438	0.2228		6.4296	0.9654
[12]	22.9430	0.0967	1.9923		6.4732	0.9651
[13]	21.3607	0.0137	1.5825		6.4302	0.9654
[14]	21.4505	3.5880	0.1367		6.3783	0.9656
[15]	0.1383	-0.1353	22.7063		6.3965	0.9707
[16]	6.4060	0.1092	0.0380		6.5054	0.9649
[17]	32.6279	-11.7095	1.3194		6.5396	0.9648
[18]	35.4102	-16.9804	2.3825		6.5871	0.9645
[19]	25.9658	0.0079	1.8180		6.4861	0.9651
[20]	30.6819	1.0633	0.0438	1.0657	6.7381	0.9637
[21]	44.0085	-1326.2916	28.0645		6.7868	0.9634
[22]	42.9266	7.8409	0.7033		6.6786	0.9640
[23]	20.9181	3.1459	0.0926	1.1304	6.3729	0.9657
[24]	21.4505	0.1367	9.3480		6.3783	0.9656
[25]	44.1298	0.0395	1.4000	0.8356	7.1371	0.9616

TABLE 6. Parameter estimates for height-diameter models for black spruce.

Model	a	b	c	d	MSE	R^2
[1]	1.0650	0.8868			3.6345	0.9651
[2]	3.2586	-11.8823			3.4830	0.9668
[3]	76.3591	81.3313			3.5027	0.9664
[4]	41.5514	0.0225			3.4925	0.9665
[5]	2.7940	0.2918			3.3299	0.9680
[6]	23.7743	-9.7182			3.6436	0.9650
[7]	0.0273	0.8868			3.6345	0.9651
[8]	0.8402	0.7263			3.7521	0.9640
[9]	24.8225	10.7421			3.5425	0.9660
[10]	4.1969	-5.9281	-0.4613		3.3328	0.9680
[11]	16.9515	10.1685	0.2232		3.5474	0.9660
[12]	22.2124	0.0729	1.4633		3.3322	0.9680
[13]	20.8565	0.0253	1.3119		3.3372	0.9679
[14]	18.6760	3.0983	0.1344		3.3918	0.9675
[15]	0.0901	0.3676	22.3022		3.3424	0.9741
[16]	2.4652	0.6413	0.0283		3.3283	0.9681
[17]	17.4475	-3.2460	1.0739		12.0102	0.8847
[18]	32.2518	-17.9185	3.4605		3.3277	0.9681
[19]	27.5359	0.0165	1.4327		3.3295	0.9681
[20]	29.7251	1.0283	0.0399	1.0815	3.3594	0.9678
[21]	42.8514	-1475.6060	33.0143		3.3409	0.9679
[22]	66.4854	5.9281	0.4613		3.3328	0.9680
[23]	8.5338	0.1268	1959.7062	8.7088	*	*
[24]	18.6759	0.1344	8.4131		3.3918	0.9675
[25]	42.0144	0.0374	0.7000	0.8187	3.7239	0.9643

NOTE: *, failure of convergence

TABLE 7. Parameter estimates for height-diameter models for white spruce.

Model	a	b	c	d	MSE	R
[1]	0.8654	0.8924			3.7859	0.9634
[2]	3.3954	-16.0789			3.6231	0.9649
[3]	90.7654	121.6496			3.4940	0.9662
[4]	49.2573	0.0151			3.4673	0.9664
[5]	5.0700	0.3599			3.1769	0.9693
[6]	28.0770	-13.9595			3.9443	0.9618
[7]	-0.0628	0.8924			3.7860	0.9634
[8]	0.6929	0.5849			4.0264	0.9610
[9]	28.9021	14.9699			3.7786	0.9634
[10]	4.5886	-6.8664	-0.4113		3.2069	0.9691
[11]	20.5713	11.7079	0.1579		3.5237	0.9659
[12]	27.3476	0.0469	1.4360		3.1387	0.9697
[13]	25.5603	0.0148	1.3081		3.1349	0.9696
[14]	22.8153	3.1887	0.0908		3.2144	0.9689
[15]	0.0593	0.3990	26.8356		3.1352	0.9752
[16]	4.1347	0.8351	0.0228		3.1704	0.9694
[17]	41.4856	-9.5447	1.0794		3.1479	0.9697
[18]	39.7715	-27.8572	5.5976		3.1430	0.9696
[19]	33.9942	0.0095	1.4114		3.1442	0.9696
[20]	26.2466	0.9428	0.0537	1.7235	3.1381	0.9697
[21]	55.7207	-3297.7051	57.5414		3.2266	0.9688
[22]	98.3565	6.8664	0.4113		3.2069	0.9690
[23]	27.6315	5.3727	0.3285	0.6044	3.1385	0.9697
[24]	22.8154	0.0908	12.7699		3.2144	0.9689
[25]	25.2774	0.0135	-0.2064	1.3381	3.1387	0.9697

TABLE 8. Parameter estimates for height-diameter models for trembling aspen.

Model	a	b	c	d	MSE	R
[1]	2.6124	0.6345			6.9288	0.9748
[2]	3.4587	-11.0518			5.8674	0.9787
[3]	44.5858	29.2468			5.8233	0.9788
[4]	29.5905	0.0481			5.6570	0.9794
[5]	1.6355	0.2152			5.6087	0.9796
[6]	30.2176	-9.3978			6.2499	0.9773
[7]	0.4170	0.6345			6.9288	0.9748
[8]	5.9319	0.5634			8.5227	0.9690
[9]	30.9617	10.1891			6.0424	0.9780
[10]	3.8994	-5.3055	-0.5611		5.6629	0.9794
[11]	23.9437	6.3854	0.1597		5.8369	0.9788
[12]	27.0190	0.0667	1.2249		5.5395	0.9799
[13]	26.4919	0.0388	1.1482		5.5359	0.9780
[14]	25.0230	2.4799	0.1080		5.5993	0.9797
[15]	0.0708	0.7053	28.0084		5.5354	0.9825
[16]	1.3488	0.4704	0.0273		5.6030	0.9797
[17]	35.2269	-7.9098	1.2043		5.5655	0.9798
[18]	35.9494	-15.6759	3.5565		5.5743	0.9798
[19]	32.3786	0.0254	1.3231		5.5642	0.9798
[20]	26.4789	0.9096	0.0749	1.5479	5.5347	0.9799
[21]	39.1200	-829.7822	19.9765		5.6412	0.9795
[22]	49.3730	5.3055	0.5611		5.6629	0.9794
[23]	27.0765	3.9064	0.3163	0.6690	5.5460	0.9799
[24]	25.0203	0.1080	8.4088		5.5993	0.9797
[25]	26.1429	0.0320	-0.4715	1.2122	5.5353	0.9799

TABLE 9. Parameter estimates for height-diameter models for white pine.

Model	a	b	c	d	MSE	R
[1]	1.6223	0.7173			9.2874	0.9634
[2]	3.5296	-16.7496			8.9616	0.9646
[3]	57.2646	59.6771			8.3574	0.9670
[4]	36.0197	0.0256			8.2685	0.9674
[5]	4.1609	0.3215			8.2483	0.9675
[6]	32.9127	-15.0604			9.3765	0.9630
[7]	0.2101	0.7173			9.2874	0.9634
[8]	3.8630	0.4791			10.8720	0.9571
[9]	33.4854	15.8737			9.1655	0.9638
[10]	4.4313	-6.2928	-0.4278		8.2086	0.9676
[11]	25.6337	7.8873	0.1129		8.9334	0.9648
[12]	31.6215	0.0367	1.1899		8.1581	0.9678
[13]	30.9183	0.0214	1.1307		8.1630	0.9678
[14]	27.3482	2.6881	0.0714		8.4184	0.9668
[15]	0.0391	0.7475	31.7029		8.1633	0.9720
[16]	2.2531	0.8256	0.0214		8.1693	0.9678
[17]	45.4612	-8.3439	1.0335		8.1544	0.9679
[18]	42.2084	-28.1327	6.5547		8.1492	0.9679
[19]	39.7993	0.0137	1.2509		8.1518	0.9679
[20]	31.9159	1.0101	0.0354	1.1426	8.1612	0.9678
[21]	49.7236	-2235.3983	42.6205		8.1963	0.9677
[22]	84.0387	6.2928	0.4278		8.2086	0.9676
[23]	35.6698	6.9436	0.5574	0.4348	8.1548	0.9679
[24]	27.3483	0.0714	13.8499		8.4184	0.9668
[25]	31.7104	0.0252	0.5168	1.0748	8.1629	0.9678

TABLE 10. Parameter estimates for height-diameter models for red pine.

Model	a	b	c	d	MSE	R
[1]	1.2209	0.8192			12.2648	0.9585
[2]	3.6773	-19.8429			11.3599	0.9615
[3]	83.8351	95.0614			11.6249	0.9606
[4]	48.1354	0.0181			11.5576	0.9609
[5]	5.3087	0.3309			11.0407	0.9626
[6]	38.1237	-18.0356			11.5785	0.9608
[7]	0.0867	0.8192			12.2698	0.9585
[8]	2.1124	0.5846			13.0609	0.9558
[9]	38.8056	18.9126			11.4667	0.9612
[10]	4.3047	-8.0952	-0.5414		11.1097	0.9625
[11]	25.9090	10.9212	0.1322		11.5925	0.9608
[12]	31.5167	0.0487	1.5772		11.0141	0.9627
[13]	29.7470	0.0115	1.3666		11.0186	0.9627
[14]	27.9577	3.2310	0.0814		11.1316	0.9623
[15]	0.0586	0.3495	31.3228		11.0187	0.9672
[16]	6.3374	0.5709	0.0223		11.0394	0.9626
[17]	47.8683	-10.7446	1.1089		11.0361	0.9627
[18]	46.1240	-28.0922	4.9393		11.0294	0.9627
[19]	37.5550	0.0067	1.5352		11.0142	0.9627
[20]	31.0864	0.9685	0.0513	1.7244	11.0208	0.9627
[21]	59.7664	-3193.4849	51.1277		11.1722	0.9622
[22]	74.0437	8.0952	0.5414		11.1097	0.9624
[23]	32.7372	5.9946	0.3481	0.5890	11.0179	0.9627
[24]	27.9577	0.0814	14.4155		11.1316	0.9623
[25]	43.0744	0.0401	2.7000	0.8010	12.5206	0.9581

TABLE 11. Parameter estimates for height-diameter models for balsam fir.

Model	a	b	c	d	MSE	R
[1]	0.8711	0.9310			2.6374	0.9503
[2]	3.1563	-11.4609			2.3956	0.9548
[3]	90.2157	108.8795			2.5227	0.9524
[4]	47.3599	0.0175			2.5142	0.9526
[5]	3.0299	0.3138			2.2793	0.9570
[6]	20.9630	-9.0597			2.5916	0.9511
[7]	-0.0599	0.9310			2.6374	0.9503
[8]	0.2569	0.7080			2.7011	0.9491
[9]	22.1399	10.1911			2.4834	0.9532
[10]	4.0996	-5.9834	-0.4703		2.2926	0.9568
[11]	15.0308	12.4049	0.2584		2.4330	0.9542
[12]	20.1136	0.0791	1.5449		2.2804	0.9570
[13]	18.7403	0.0234	1.3670		2.2858	0.9569
[14]	16.8565	3.2987	0.1465		2.3165	0.9564
[15]	0.1078	0.1975	19.8436		2.2889	0.9674
[16]	2.9831	0.6369	0.0321		2.2805	0.9570
[17]	27.1559	-9.8022	1.2656		2.2774	0.9571
[18]	29.7533	-17.4797	3.2353		2.2756	0.9572
[19]	24.3867	0.0155	1.4942		2.2763	0.9571
[20]	20.1406	1.0016	0.0787	1.5366	2.2816	0.9570
[21]	40.6687	-1440.2357	33.9715		2.3039	0.9566
[22]	60.3138	5.9834	0.4703		2.2926	0.9568
[23]	20.9429	59.6261	-0.2533	-1.7346	*	*
[24]	16.8565	0.1465	8.1485		2.3165	0.9564
[25]	19.8270	0.0315	0.5543	1.2380	2.2842	0.9570

NOTE: *, failure of convergence

TABLE 12. Parameter estimates for height-diameter models for yellow birch.

Model	a	b	c	d	MSE	R
[1]	3.7404	0.4676			5.6327	0.9689
[2]	3.1688	-7.4706			3.6296	0.9798
[3]	27.5416	13.9089			3.7292	0.9793
[4]	20.9619	0.0804			3.4809	0.9806
[5]	0.8049	0.1801			3.5530	0.9803
[6]	22.5812	-6.0075			3.9566	0.9780
[7]	0.5729	0.4676			5.6372	0.9687
[8]	8.3529	0.3176			8.1345	0.9548
[9]	23.1629	6.7082			3.7633	0.9791
[10]	3.3410	-4.1390	-0.6789		3.6082	0.9800
[11]	19.6354	4.9018	0.1854		3.7910	0.9789
[12]	20.7576	0.0858	1.0532		3.4822	0.9807
[13]	20.7055	0.0759	1.0345		3.4817	0.9808
[14]	20.0279	2.1303	0.1323		3.5949	0.9801
[15]	0.0864	0.9244	22.0426		3.4825	0.9837
[16]	0.7048	0.3786	0.0398		3.5591	0.9803
[17]	23.7914	-6.9676	1.3589		3.5536	0.9803
[18]	24.9856	-9.0828	2.0794		3.5515	0.9803
[19]	23.4921	0.0530	1.2774		3.5292	0.9804
[20]	20.8884	1.0420	0.0802	0.9280	3.4886	0.9807
[21]	25.6560	-271.9318	9.6165		3.5720	0.9802
[22]	28.2458	4.1391	0.6790		3.6082	0.9800
[23]	21.1572	4.0751	0.4988	0.5851	3.5056	0.9806
[24]	20.0279	0.1323	5.7158		3.5949	0.9801
[25]	20.8157	0.0851	0.2627	0.9943	3.4895	0.9807

TABLE 13: Parameter estimates for height-diameter models for balsam poplar.

Model	a	b	c	d	MSE	R
[1]	2.9729	0.5868			5.8699	0.9843
[2]	3.4970	-12.2283			4.5586	0.9878
[3]	43.0973	28.3846			4.6925	0.9875
[4]	29.4500	0.0470			4.5127	0.9880
[5]	1.8605	0.2271			4.4124	0.9882
[6]	31.8390	-10.7670			4.7933	0.9872
[7]	0.4732	0.5868			5.8698	0.9843
[8]	7.4714	0.4795			7.6355	0.9796
[9]	32.3997	11.4674			4.6664	0.9875
[10]	3.7808	-6.0371	-0.6417		4.4967	0.9881
[11]	24.4648	6.6300	0.1492		4.5924	0.9878
[12]	27.0418	0.0670	1.3050		4.3803	0.9884
[13]	26.5287	0.0331	1.1865		4.3820	0.9884
[14]	25.4561	2.5542	0.1021		4.4116	0.9883
[15]	0.0719	0.6223	28.0183		4.3750	0.9897
[16]	2.1710	0.4211	0.2848		4.4270	0.9882
[17]	34.8937	-8.6813	1.2310		4.4055	0.9883
[18]	35.7474	-15.6913	3.1557		4.4142	0.9883
[19]	31.4760	0.0199	1.4140		4.3867	0.9884
[20]	26.5002	0.8622	0.0764	1.8076	4.3904	0.9884
[21]	38.3382	-790.2765	19.0039		4.4905	0.9881
[22]	43.8525	6.0371	0.6417		4.4967	0.9881
[23]	27.0923	3.9428	0.2897	0.6913	4.3807	0.9885
[24]	25.4561	0.1021	9.1851		4.4116	0.9883
[25]	26.3999	0.0303	-0.2292	1.2149	4.4001	0.9884

Figures

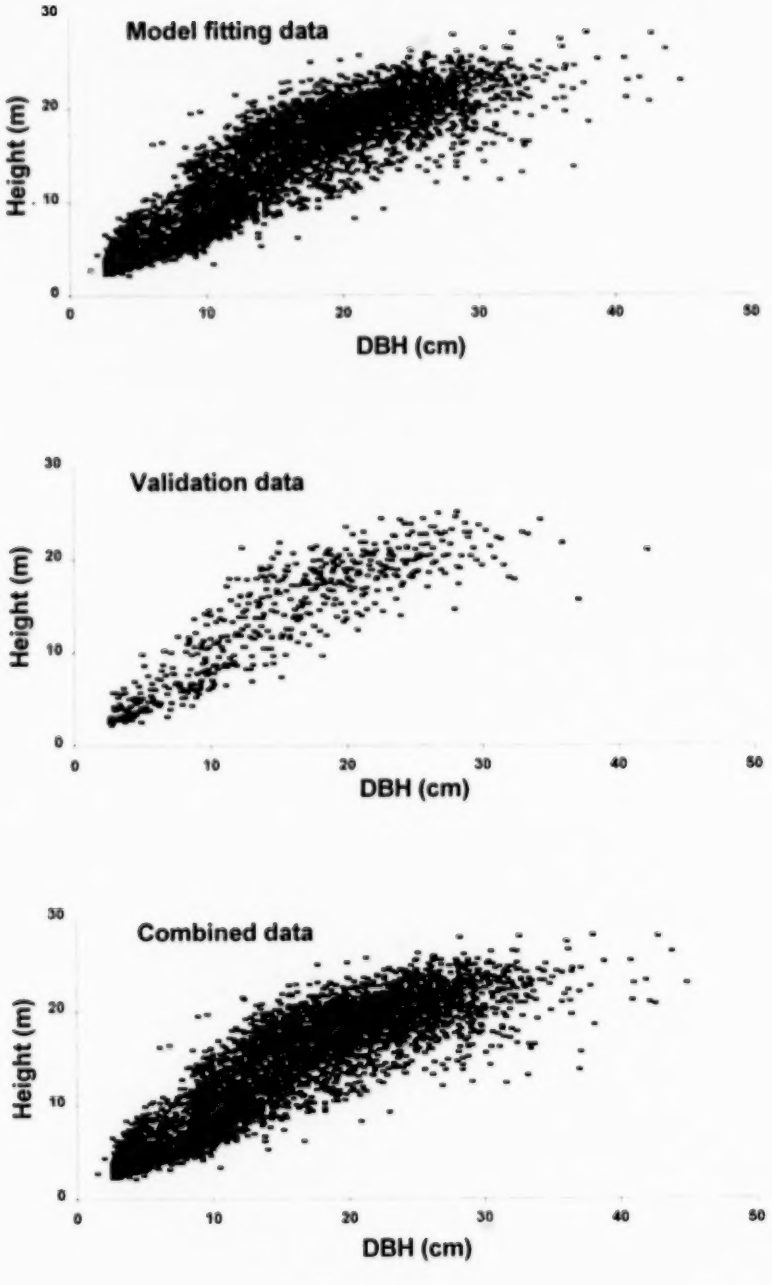


FIGURE 1. Total height (HT) plotted against diameter at breast (DBH) for jack pine.

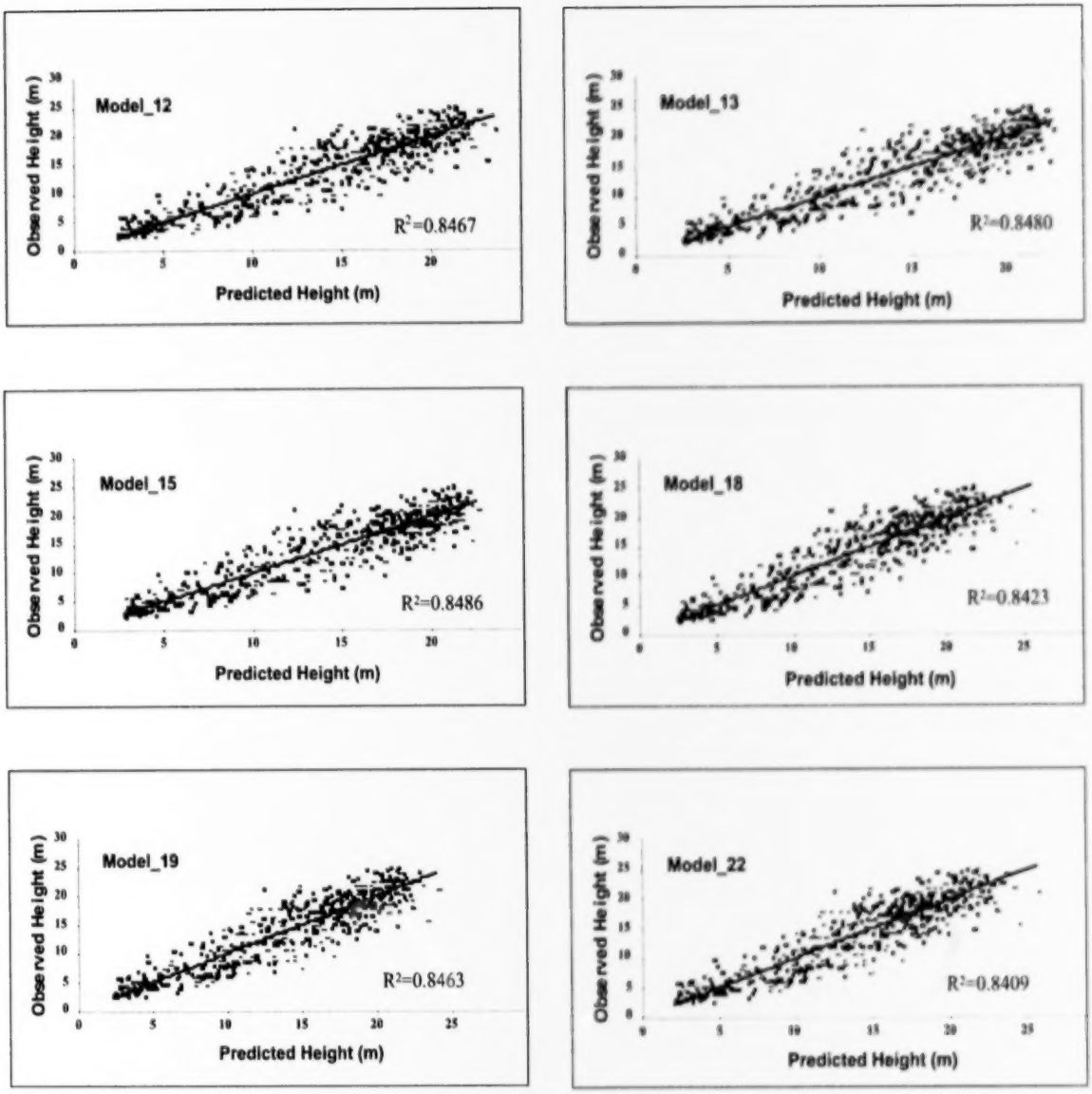


FIGURE 2. Observed vs. predicted tree heights for the validation data set for jack pine. The diagonal line presents cases where observed height equals predicted height.

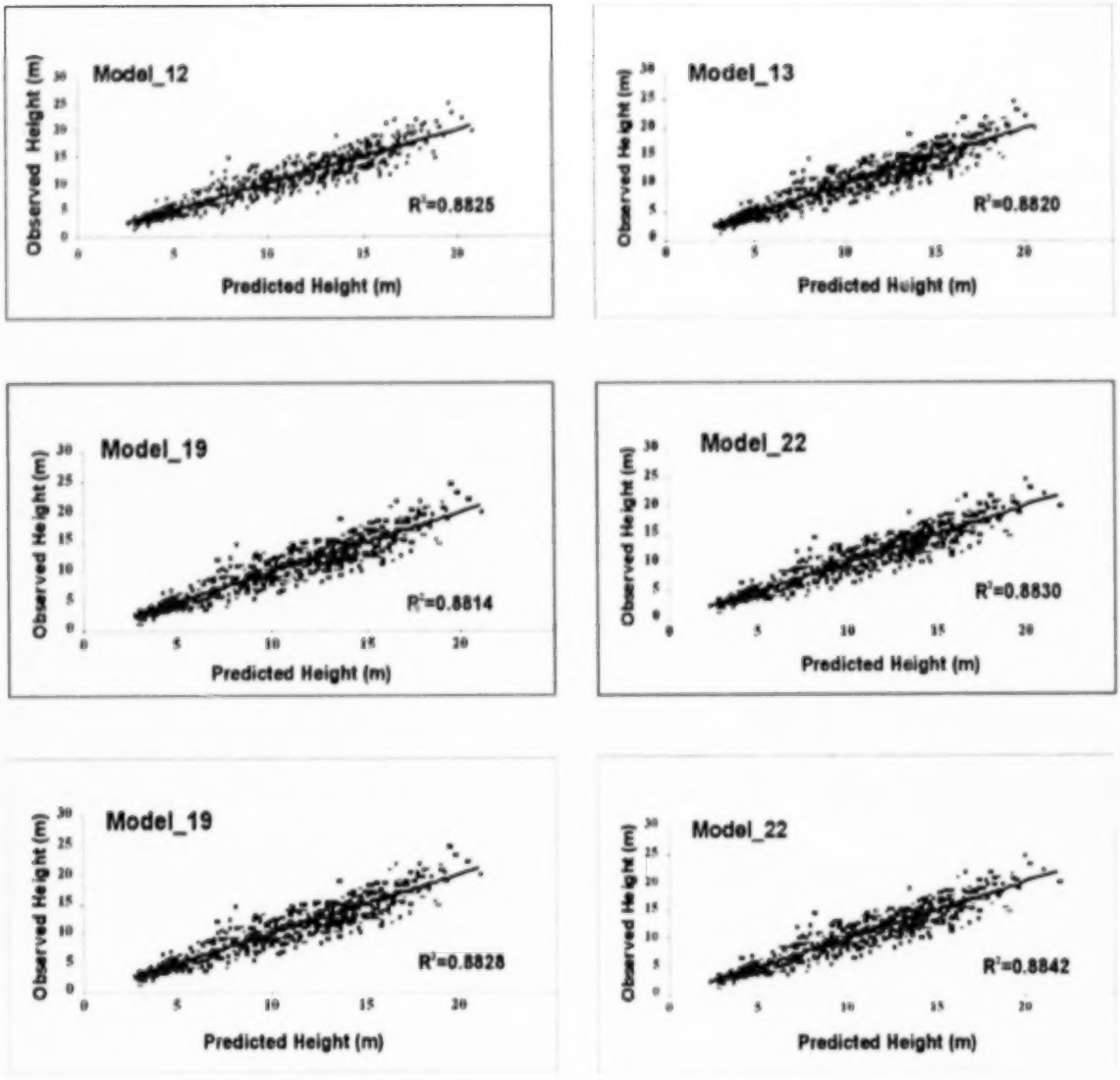
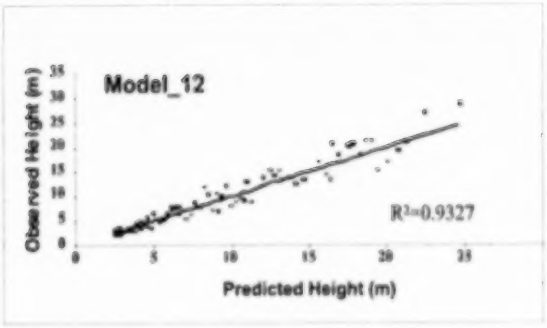
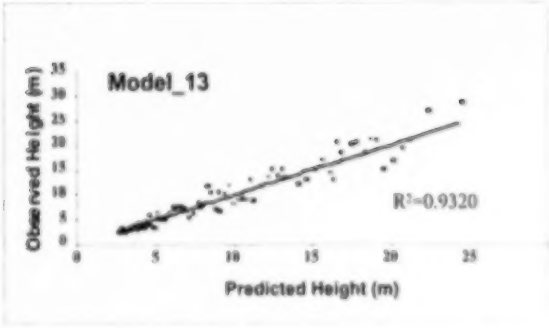
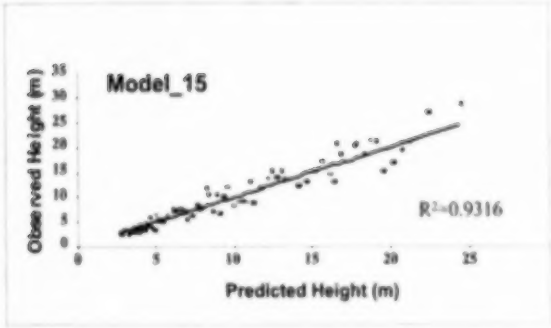
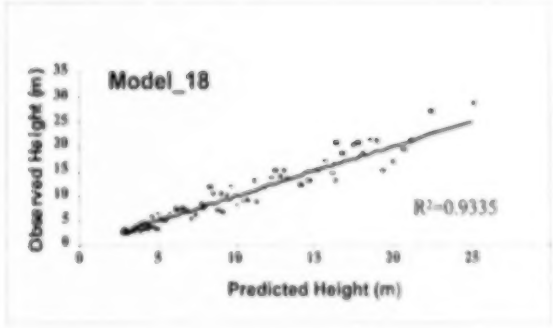


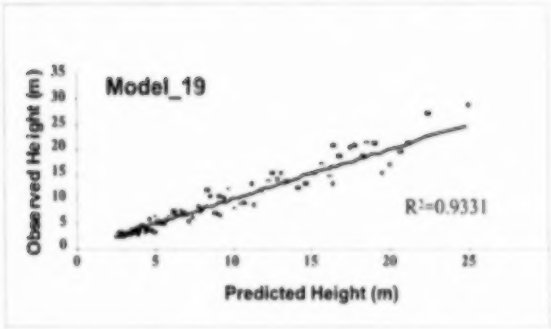
FIGURE 3. Observed vs. predicted tree heights for the validation data set for black spruce. The diagonal line presents cases where observed height equals predicted height.











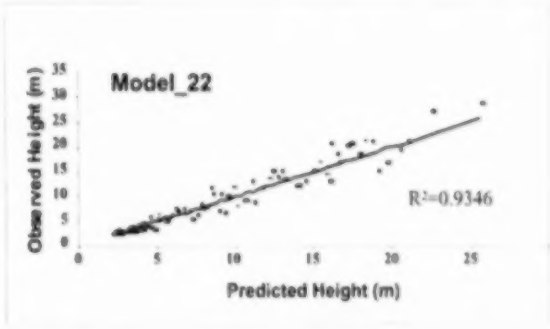


FIGURE 4. Observed vs. predicted tree heights for the validation data set for white spruce. The diagonal line presents cases where observed height equals predicted height.

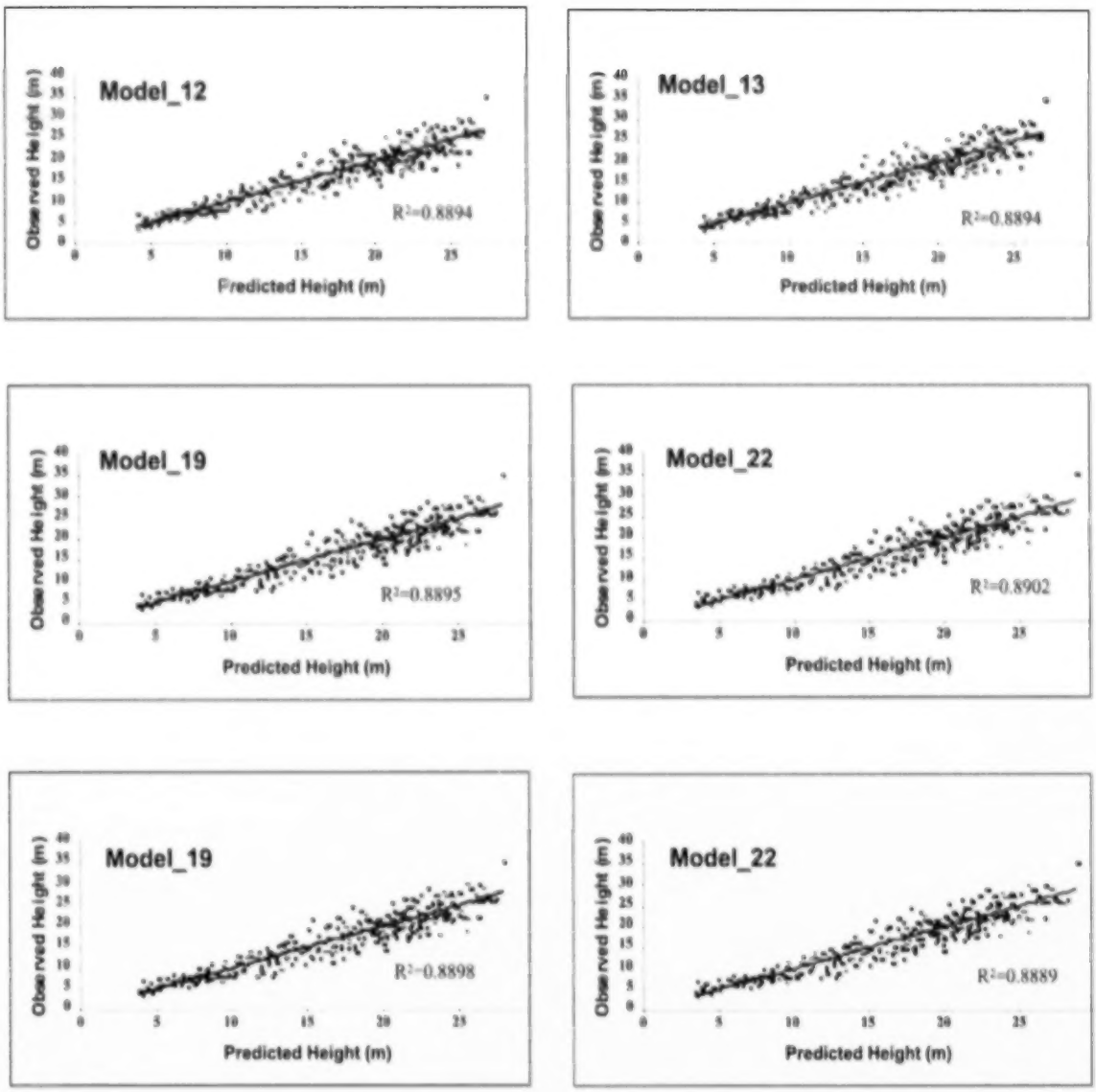


FIGURE 5. Observed vs. predicted tree heights for the validation data set for trembling aspen. The diagonal line presents cases where observed height equals predicted height.

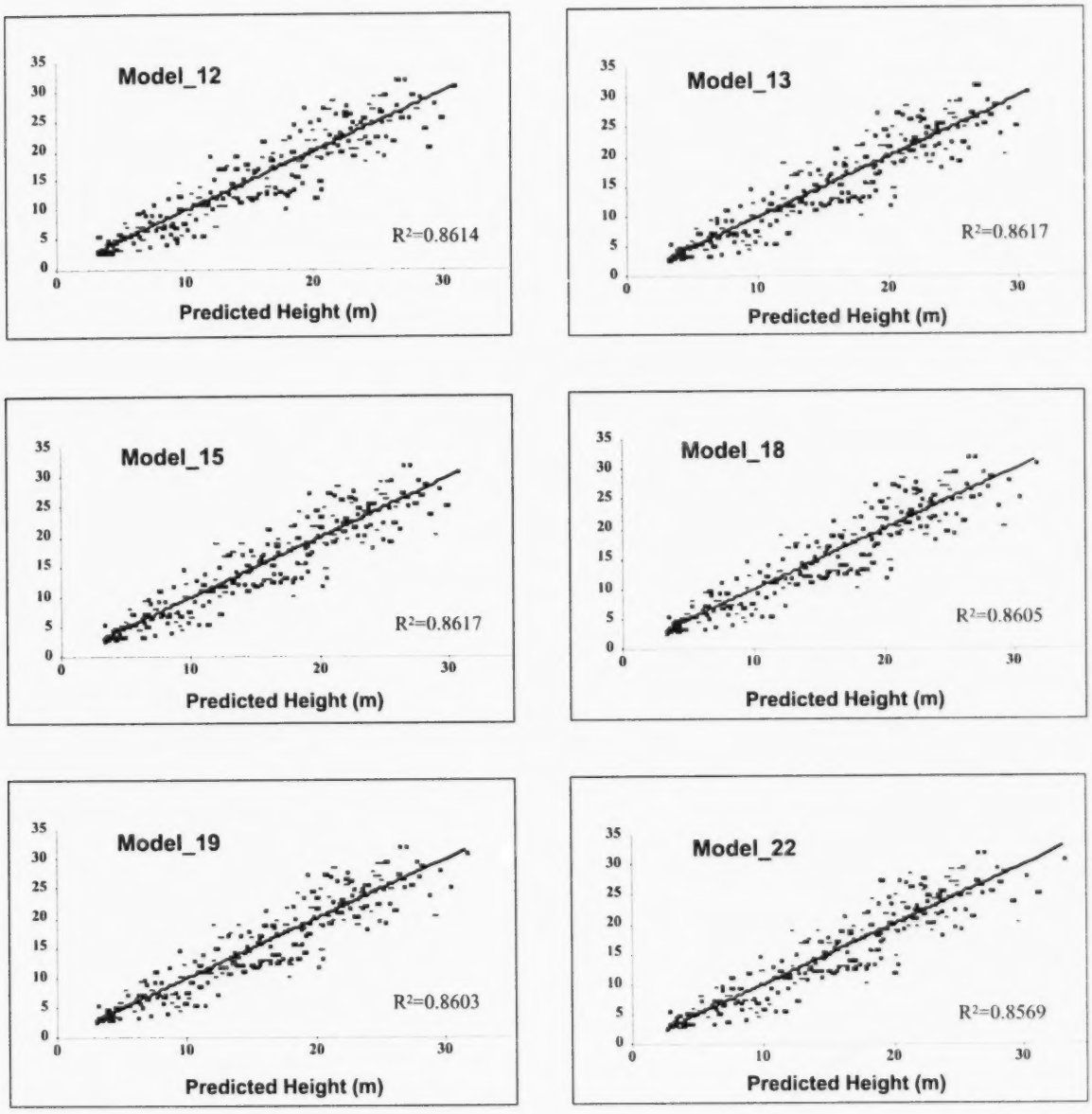


FIGURE 6. Observed vs. predicted tree heights for the validation data set for white pine. The diagonal line presents cases where observed height equals predicted height.

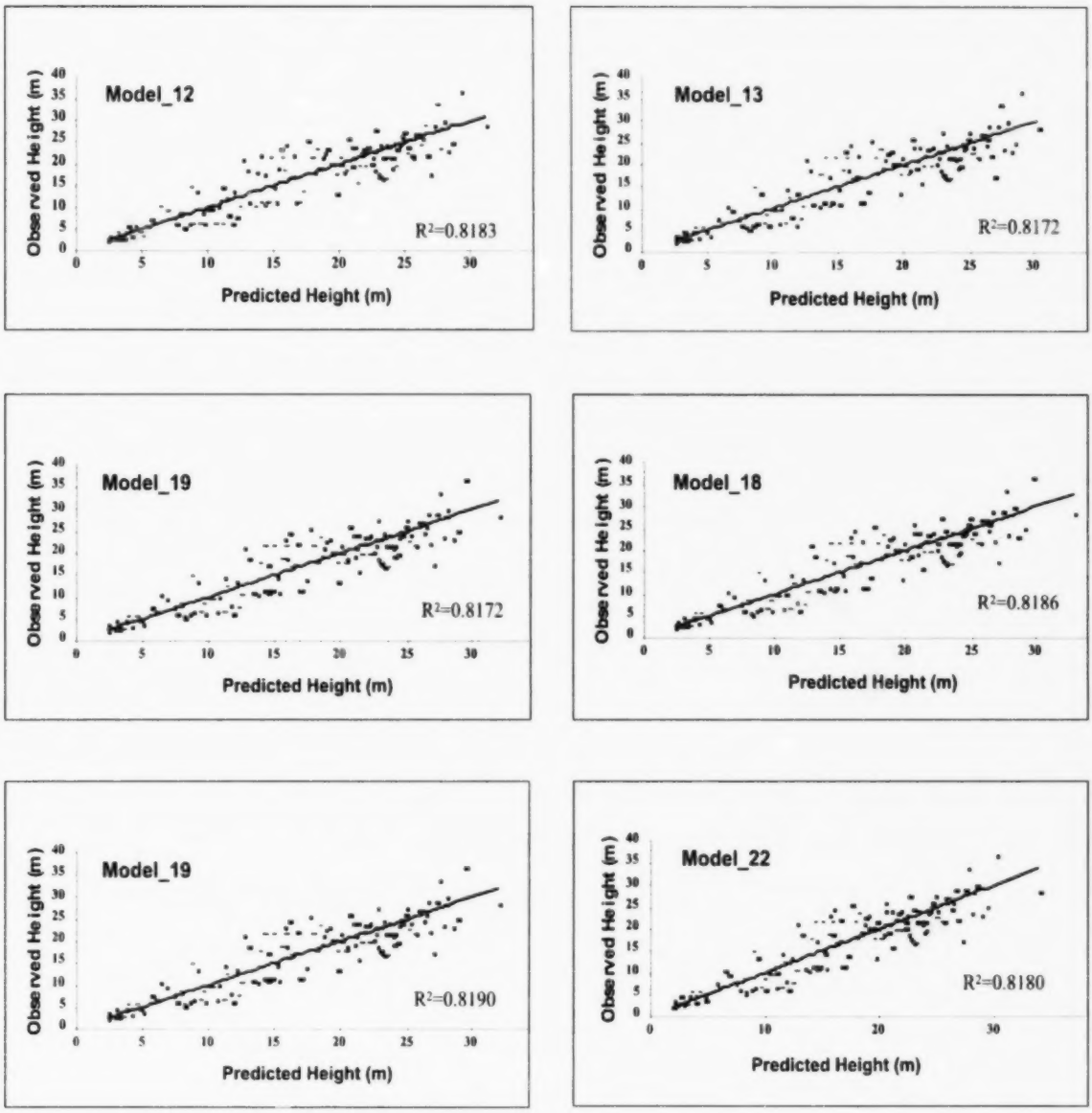


FIGURE 7. Observed vs. predicted tree heights for the validation data set for red pine. The diagonal line presents cases where observed height equals predicted height.

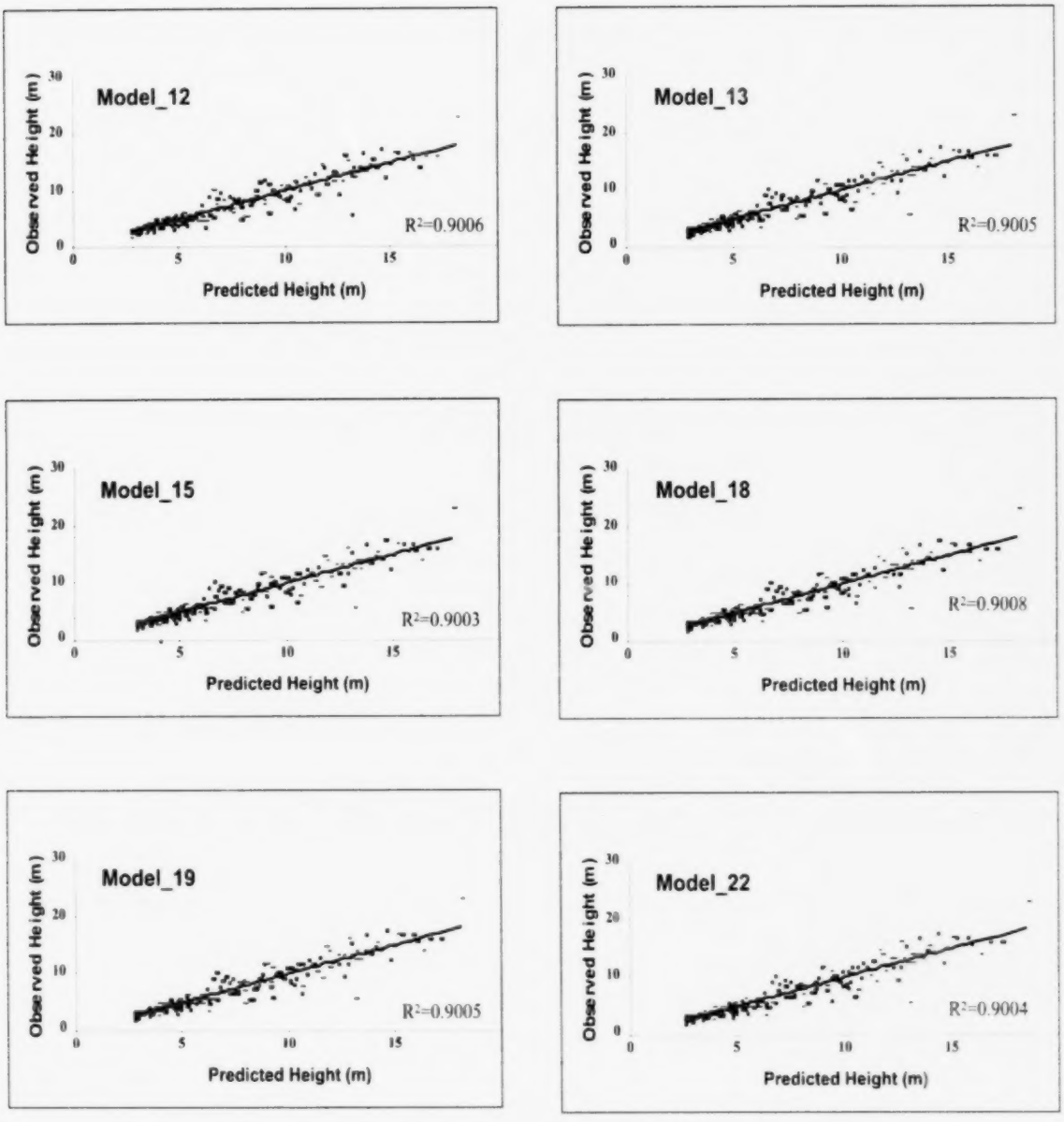


FIGURE 8. Observed vs. predicted tree heights for the validation data set for balsam fir. The diagonal line presents cases where observed height equals predicted height.

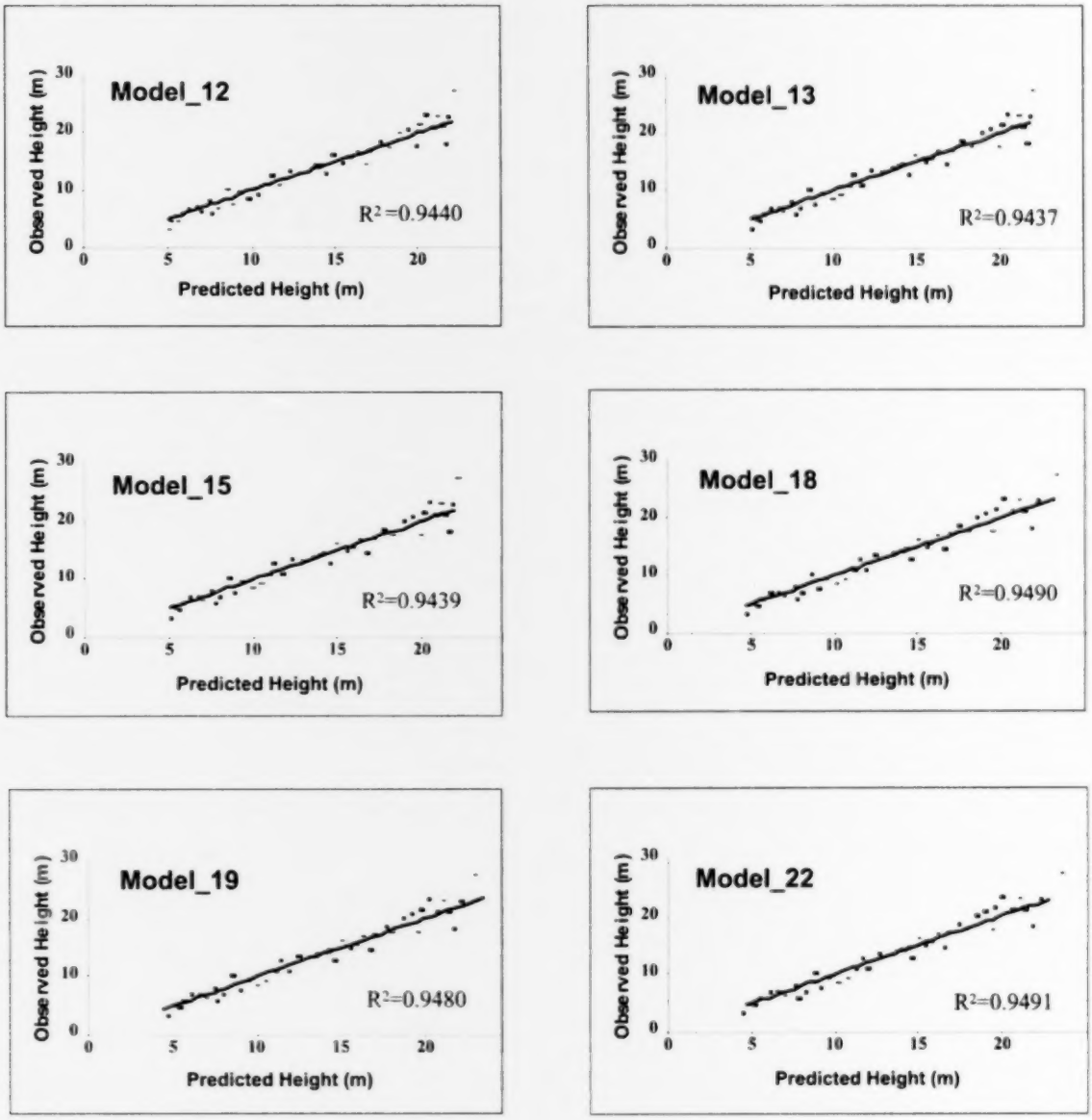
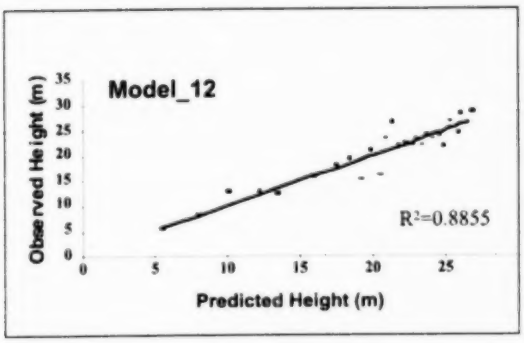
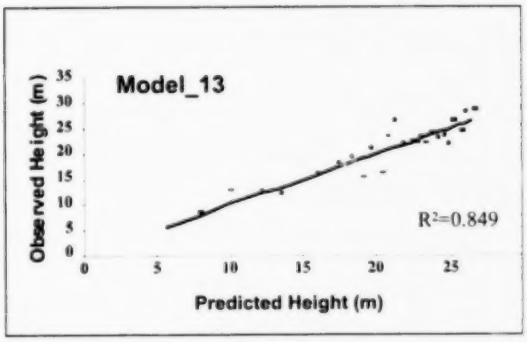
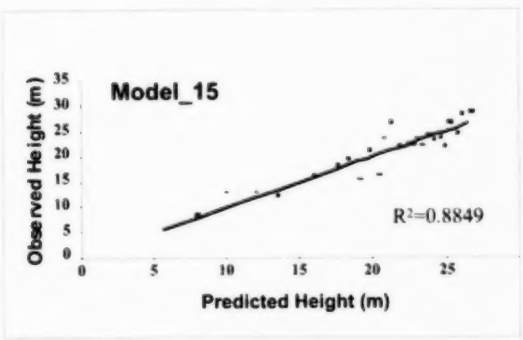
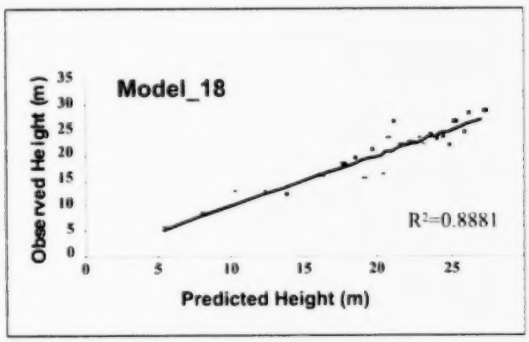


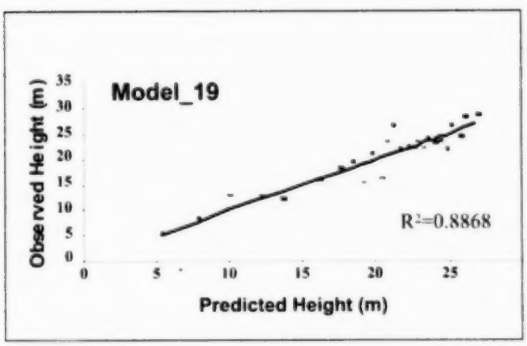
FIGURE 9. Observed vs. predicted tree heights for the validation data set for yellow birch. The diagonal line presents cases where observed height equals predicted height.











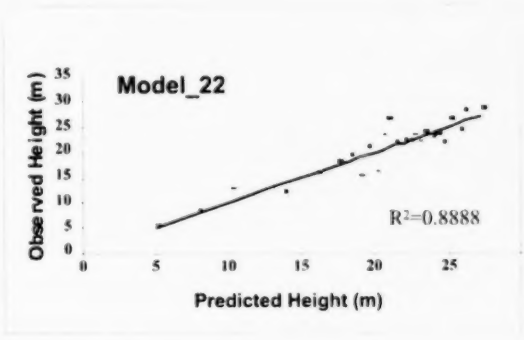


FIGURE 10. Observed vs. predicted tree heights for the validation data set for balsam poplar. The diagonal line presents cases where observed height equals predicted height.

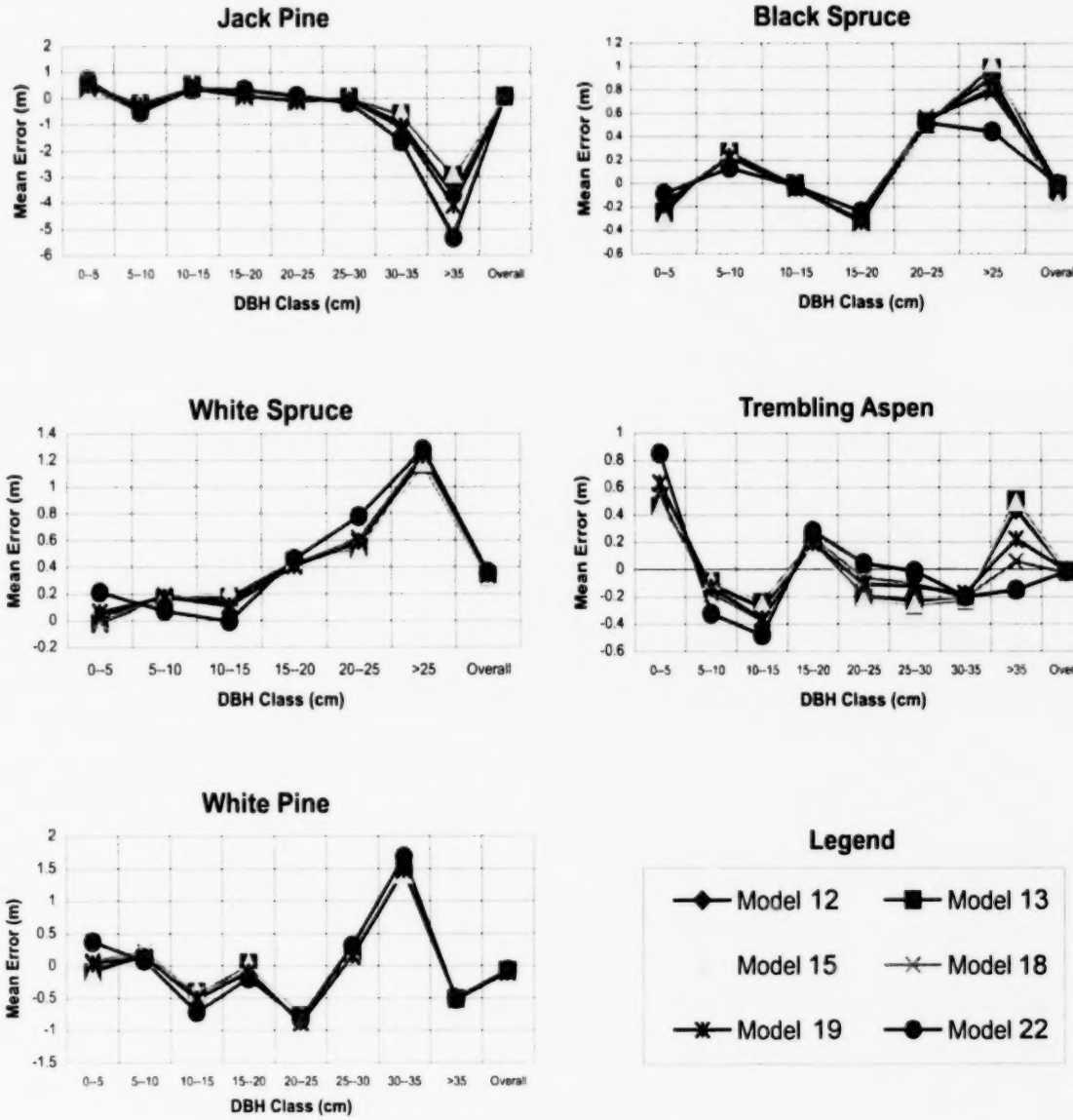


FIGURE 11. Average prediction errors from 6 tree height-diameter models [12], [13], [15], [18], [19] and [22] for the 5-cm DBH classes for 9 tree species in Ontario's boreal forests. *Overall* represents mean predicted error across all DBH classes.

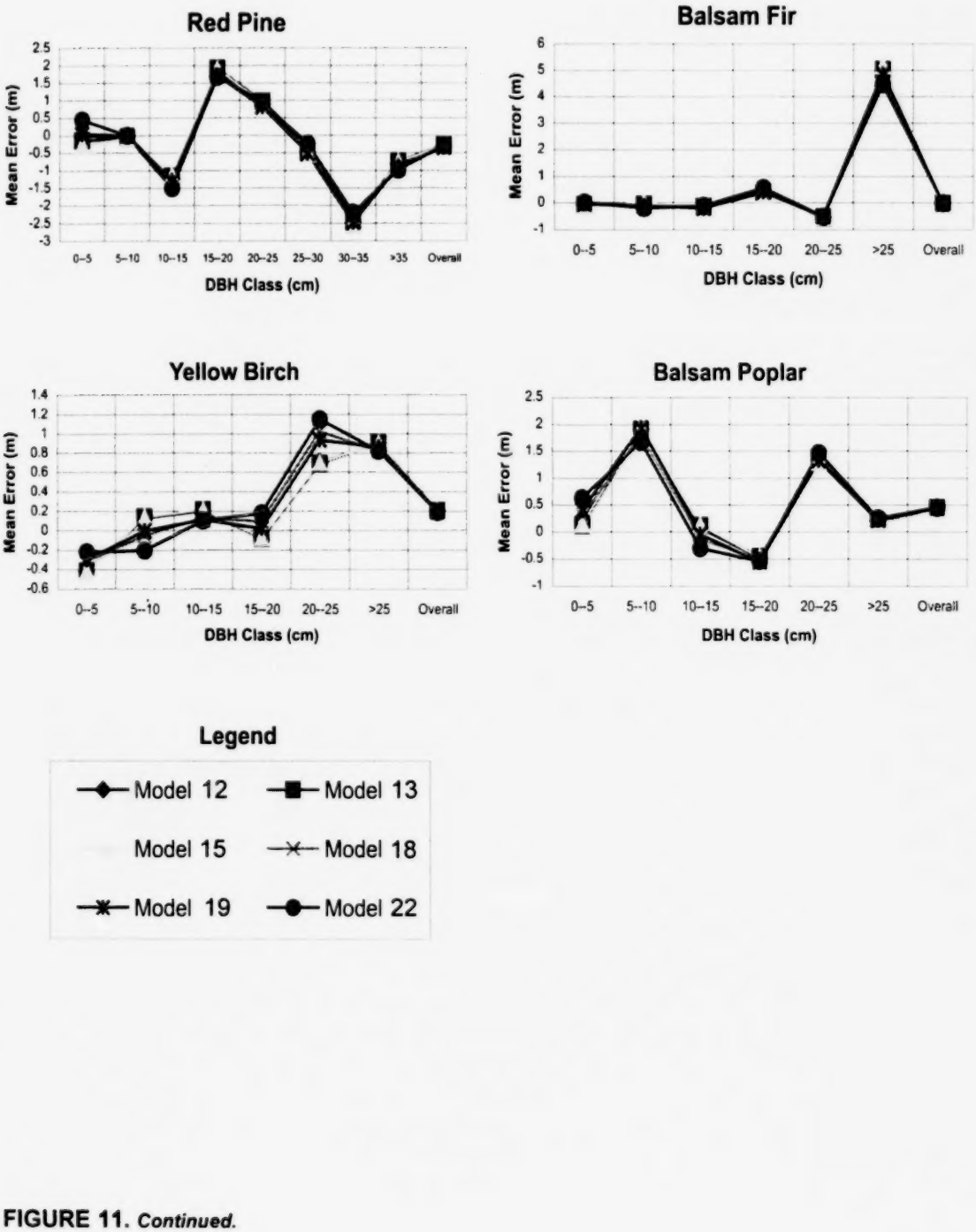


FIGURE 11. ***Continued.***

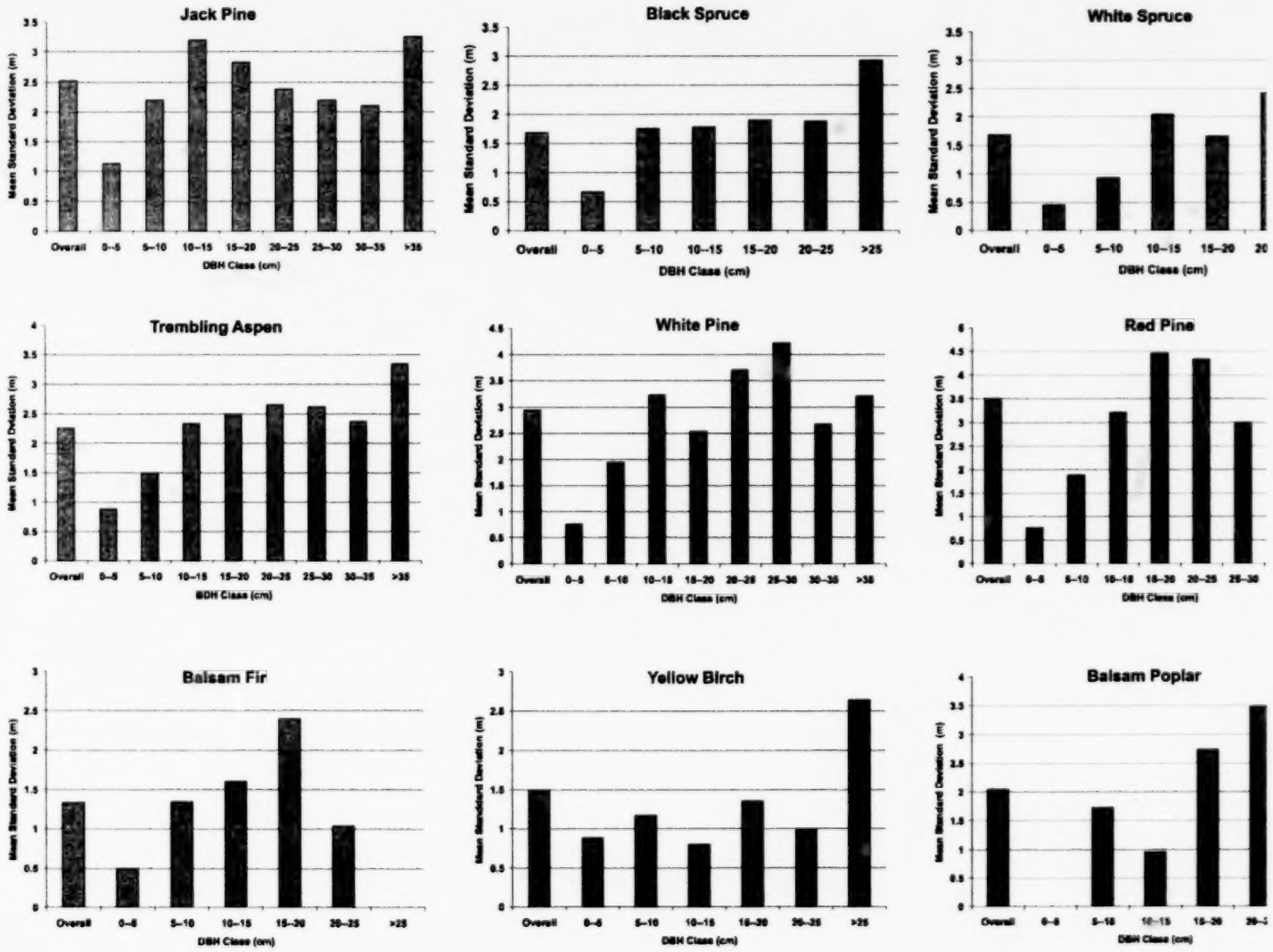


FIGURE 12. Average mean standard deviation of prediction errors from 6 tree height-diameter models [12], [13], [15], [18], [19] and [22] for the 5-cm DBH classes for 9 tree species in Ontario's boreal forests. *Overall* represents mean standard deviation of predicted error across all DBH classes. The average mean standard deviation of prediction errors was not computed if the number of validation trees was 2 or less for any DBH class (e.g., balsam fir).

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